LONG-TERM ENERGY CONSTRAINTS AND POWER CONTROL IN COGNITIVE RADIO NETWORKS

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ABSTRACT

When a long-term energy constraint is imposed to a transmitter, the average energy-efficiency of a transmitter is, in general, not maximized by always transmitting. In a cognitive radio context, this means that a secondary link can re-exploit the non-used time-slots. In the case where the secondary link is imposed to generate no interference on the primary link, a relevant issue is therefore to know the fraction of time-slots available to the secondary transmitter, depending on the system parameters. On the other hand, if the secondary transmitter is modeled as a selfish and free player choosing its power control policy to maximize its average energy-efficiency, resulting primary and secondary signals are not necessarily orthogonal and studying the corresponding Stackelberg game is relevant to know the outcome of this interactive situation in terms of power control policies.

***Index Terms*—** Cognitive radio, Energy-efficiency, Power control, Primary user, Secondary user, Stackelberg games.

1. INTRODUCTION

One of the ideas of cognitive radio is to allow some wireless terminals, especially transmitters, to sense their environment in terms of used spectrum and to react to it dynamically. The cognitive radio paradigm [1] has become more and more important to the wireless community since the release of the FCC report [2]. Indeed, cognitive radio corresponds to a good way of tackling the crucial problem of spectrum congestion and increasing spectral efficiency. More recently, the main actors of the telecoms industry, namely carriers, manufacturers, and regulators have also realized the importance of energy aspects in wireless networks (see e.g., [3]) both at the network infrastructure and mobile terminal sides. There are many reasons for this and we will not provide them here. As far as this paper is concerned, the goal is to study the influence of long-term energy constraints (e.g., the limited battery life typically) on power control in networks where cognitive radios are involved. The performance criterion which is considered for the terminal is derived from the one introduced by Goodman and Mandayam in [4]. Therein, the authors propose a distributed power control scheme for frequency non-selective block fading multiple access channels. For each block, a terminal aims at maximizing its individual energy-efficiency namely, the number of successfully decoded bits at the receiver per Joule consumed at the transmitter. Although, a power control maximizing such a performance metric is called energy-efficient, it does not take into account possible long-term energy constraints. Indeed, in [4] and related references (e.g., [5][6]), the terminals always transmit, which amounts to considering no constraints on the available (average) energy. The goal of the present work is precisely to see how energy constraints modify power control policies in a single-user channel and in a cognitive radio channel. For the sake of simplicity, time-slotted communications are assumed.

The paper is organized in two main parts. In Sec. 3 a single-user channel is considered. It is shown that maximizing an average energy-efficiency under a long-term energy constraint leads the terminal to not transmit on certain blocks. The probability that the terminal does not transmit is lower bounded. In a setting where a primary transmitter has to control its power under energy-constraint, this probability matters since it corresponds to the fraction of available time-slots which are re-exploitable by a secondary (cognitive) transmitter. In Sec. 3, the single-user channel model is sufficient since the secondary link has to meet a zero interference constraint (it can only exploit non-used time-slots). In Sec. 4, the secondary transmitter is assumed to be free to use all the time-slots. The technical difference between the primary and secondary transmitters is that the former has to choose its power level in the first place while the latter observes this level and react to it. The suited interaction model is therefore a Stackelberg game [7] where the primary and secondary transmitters are respectively the leader and follower of the game. Sec. 5 provides numerical results which allow us to validate some derived results and compare the two cognitive settings (depending whether the secondary transmitter can generate non-orthogonal signals).

2. GENERAL SYSTEM MODEL

In the whole paper the goal is to study a system comprising two transmitter-receiver pairs. The signal model under consideration can be described by a frequency non-selective block fading channel. The signals received by the two receivers write as:

$$\begin{array}{rcl} y_1 & = & h_{11}x_1 + h_{21}x_2 + z_1 \\ y_2 & = & h_{22}x_2 + h_{12}x_1 + z_2 \end{array} . \quad (1)$$

The channel gain of the link ij namely, h_{ij} is assumed to be constant over each block or time-slot. The quantity $g_{ij} = |h_{ij}|^2$ is assumed to be a continuous random variable having independent realizations and distributed according to the probability density function $\phi_{ij}(g_{ij})$. The reception noises are zero-mean complex white Gaussian noises with variance σ^2. The instantaneous power of the transmitted signal x_i on time-slot t is given by

$$p_i(t) = \frac{1}{N}\sum_{n=1}^{N} |x(n)|^2 \quad (2)$$

where n is the symbol index and N the number of symbols per time-slot. For simplicity, transmissions are assumed to be time-slotted.

Transmitter 1 (resp. 2), receiver 1 (resp. 2), link 11 (resp. 22) will be respectively called primary (resp. secondary) transmitter, primary (resp. secondary) receiver, and (resp. secondary) primary link. The main technical difference between the primary and the secondary links is that the secondary transmitter can observe the power levels chosen by the primary transmitter but the converse does not hold. In this paper, two scenarios are investigated:

- Scenario 1 (Sec. 3): the secondary transmitter is imposed to meet a zero-interference constraint on the primary link. Since the primary and secondary signals are orthogonal, everything happens for the transmitter 1 as if it was transmitting over a single-user channel.
- Scenario 2 (Sec. 4): this time, the secondary transmitter can use all the time-slots and not only those not exploited by the primary link. Primary and secondary signals are therefore not orthogonal in general. In this framework, for each time-slot, the primary transmitter chooses its power level and is informed that the secondary will observe and react to it in a rational manner. A Stackelberg game formulation is proposed to study this interactive situation.

3. WHEN PRIMARY AND SECONDARY SIGNALS ARE ORTHOGONAL

3.1. Optimal power control scheme for the primary transmitter

From the primary point of view, there is no interference and the signal-to-noise plus interference ratio (SINR) coincides with the signal-to-noise ratio (SNR):

$$\mathrm{SNR}(p_1(g_{11})) = \frac{g_{11}p_1(g_{11})}{\sigma^2}. \quad (3)$$

When using the notation $p_1(g_{11})$ instead of $p_{11}(t)$ we implicitly make appropriate ergodicity assumptions on g_{11}. The main purpose of this section is precisely to determine the optimal control function $p_1(g_{11})$ in the sense of the long-term energy efficiency, which is defined as follows:

$$u_1(p_1(g_{11})) = R_1 \int_0^{+\infty} \phi_{11}(g_{11}) \frac{f(\mathrm{SNR}(p_1(g_{11})))}{p_1(g_{11})} \mathrm{d}g_{11} \quad (4)$$

where R_1 is the transmission rate and f is an efficiency function representing the packet success rate $f : \mathbb{R}^+ \to [0,1]$. The function f is assumed to possess the following properties:

1. f is non-decreasing, C^2 differentiable, $f(0) = 0$, $\lim\limits_{x\to+\infty} f(x) = 1$ and there exists a unique inflection point x_0 for f.
2. f' is non-negative, $f'(0) = \lim\limits_{x\to+\infty} f'(x) = 0$. f' reaches its maximum for x_0.
3. f'' is non-negative over $[0, x_0]$, negative over $[x_0, +\infty[$. $f^{(2)}(0) = 0$, $\lim\limits_{x\to+\infty} f''(x) = 0^-$.

These properties are verified by the two typical efficiency functions available in the literature:

$$f_a(x) = \left\{ \begin{array}{c} e^{-\frac{a}{x}} \ \ \forall x > 0 \\ 0 \text{ if } x = 0 \end{array} \right. \quad (5)$$

and

$$f_M(x) = \left(1 - e^{-x}\right)^M \ \ \forall x \geq 0. \quad (6)$$

The function f_a, $a \geq 0$ has been introduced in [8][9] and corresponds to the case where the efficiency function equals one minus the outage probability. On the other hand, f_M, $M \in \mathbb{N}^*$, corresponds to an empirical approximation of the packet success rate which was already used in [4].

Compared to references [4][5][6], note that the user's utility is the average energy-efficiency and not the instantaneous energy-efficiency. This allows one to take into account the following energy constraint:

$$T \int_0^{+\infty} \phi_{11}(g_{11}) p_1(g_{11}) \mathrm{d}g_{11} \leq E_1 \quad (7)$$

where T is the time-slot duration and E_1 is the available energy for terminal 1. In order to find the optimal solution(s) for the power control schemes, let us consider the Lagrangian L_{u_1}. It writes as:

$$\begin{aligned} L_{u_1} = & R_1 \int_0^{+\infty} \phi_{11}(g_{11}) \frac{f(\mathrm{SNR}(p_1(g_{11})))}{p_1(g_{11})} \mathrm{d}g_{11} \\ & - \lambda(T \int_0^{+\infty} \phi_{11}(g_{11}) p_1(g_{11}) \mathrm{d}g_{11} - E_1). \end{aligned} \quad (8)$$

It is ready to show that the optimal instantaneous signal-to-noise ratio (3) has to be the solution of $\frac{\partial L_{u_1}}{\partial p_1(g_{11})} = 0$:

$$x f'(x) - f(x) = \frac{\lambda T \sigma^4}{R_1 g_{11}^2} x^2. \quad (9)$$

Solving the above equation amounts to finding the zeros of $F(x) = x f'(x) - f(x) - \frac{\lambda T \sigma^4}{R_1 g_{11}^2} x^2$. We have that F is C^1 differentiable, $F(0) = 0$, $\lim\limits_{x\to+\infty} F(x) = -\infty$, and

$$F'(x) = x f^{(2)}(x) - 2\frac{\lambda T \sigma^4}{R_1 g_{11}^2} x. \quad (10)$$

Then, $\exists \epsilon,\ \forall x \in]0, \epsilon],\ F'(x) < 0$. Considering the sign of F', given the particular form of $f^{(2)}$, two cases have to be considered.

- If $\forall x$, $f''(x) \leq 2\frac{\lambda T \sigma^4}{R_1 g_{11}^2}$, F' is negative or null and F is decreasing. Then 0 is the only zero for F.

- If $\exists(x_1, x_2),\ x_1 < x_2$ st $f''(x_1) = f''(x_2) = \frac{\lambda T \sigma^4}{R_1 g_{11}^2}$, and F' non-negative over $[x_1, x_2]$. F decreases over $[0, x_1]$, increases over $[x_1, x_2]$ and decreases over $[x_2, +\infty[$. Then F may have zero, one or two zeros different from 0.

If F has one zero, it is 0 and 0 is the maximum for L_{u_1}. If F has two zeros: 0 and x_0', L_{u_1} is decreasing and 0 is the maximum for L_{u_1}. If F has three zeros: 0, x_1' and x_2', L_{u_1} decreases over $[0, x_1']$, increases over $[x_1', x_2']$ and decreases over $[x_2', +\infty[$. The maximum for L_{u_1} is then 0 or x_2.

Assume $\mathrm{SNR}^*_{\lambda_{E_1}}(g)$ is the greatest solution of equation (9). Then an optimal power control scheme is given by:

$$p_1^*(g_{11}) = \frac{\sigma^2}{g_{11}} \mathrm{SNR}^*_{\lambda_{E_1}}(g_{11}) \tag{11}$$

with $\mathrm{SNR}^*_{\lambda_{E_1}}(g_{11}) \geq 0$. Since E_1 is fixed, the methodology consists in determining λ_{E_1}, then a solution of (9) is determined numerically. Note that λ_{E_1} is in bits/Joule^2. It can be interpreted as a minimal number of bits to transmit for 1 Joule^2. The higher λ_{E_1} is, the better the channel should be to be used.

Remark (Capacity of fast fading channels). The proposed analysis is reminiscent to the capacity determination of fast fading single-user channels [10]. Two important differences between this and our analysis are worth being emphasized. First, mathematically, the optimization problem under study is more general than the one of [10]. Indeed, if one makes the particular choice $f(\mathrm{SNR}(p_1(g_{11}))) = p_1 \log(1 + \mathrm{SNR}(p_1(g_{11})))$, the optimal SNR is given by $\mathrm{SNR}^*(p_1(g_{11})) = \frac{g_{11}}{\lambda_{E_1}\sigma^2} - 1$, which corresponds to a water-filling solution (the SNR has to be non-negative). Second, the physical interpretation of the average utility is different from the fast fading case. In the fast fading case, the power control is updated at the symbol rate whereas in our case, it is updated at the time-slot frequency namely, $\frac{1}{T}$. Indeed, in power control problems, what is updated is the average power over a block or time-slot and assuming an average power constraint over several blocks or time-slots generally does not make sense. However, from an energy perspective introducing an average constraint is relevant. This comment is a kind of subtle and characterizes our approach.

3.2. Time-slot occupancy probability

As shown in the preceding section, time-slots are not used by the primary link when the solution $\mathrm{SNR}^*_{\lambda_{E_1}}(g_{11})$ is negative. Therefore, the probability that this event occurs corresponds to the probability of having a free time-slot for the secondary link. It is thus relevant to evaluate $\Pr[\mathrm{SNR}^*_{\lambda_{E_1}}(g_{11}) \leq 0]$. At first glance, explicating this probability does not seem to be trivial. However, one can see from the preceding section that if $\max f'' \leq 2\frac{\lambda T \sigma^4}{R_1 g_{11}^2}$, the function F has no non-negative solutions except from 0, in which case there is no power allocated to channel g_{11}. Based on this observation, the following lower bound arises:

$$\Pr\left[\max f'' \leq 2\frac{\lambda T \sigma^4}{R_1 g_{11}^2}\right] \leq \Pr[\mathrm{SNR}^*_{\lambda_{E_1}}(g_{11}) \leq 0]. \tag{12}$$

Many simulations have shown that this lower bound is reasonably tight, one of them is provided in the simulation section; what matters in this paper is to show that the fraction of available time-slots can be significant and the proposed lower bound ensures to achieve at least the corresponding performance. To conclude on this point, note that in the case where $f(\mathrm{SNR}(p_1(g_{11}))) = p_1 \log(1 + \mathrm{SNR}(p_1(g_{11})))$, the probability of having a free time-slot for the secondary link can be easily expressed and is given by:

$$\Pr\left[\mathrm{SNR}^*_{\lambda_{E_1}}(g_{11}) \leq 0\right] = 1 - e^{-\frac{\lambda_{E_1}\sigma^2}{\overline{g}_{11}}} \tag{13}$$

where $\overline{g}_{11} = E(g_{11})$. A similar analysis has been made to design a Shannon-rate efficient interference alignment technique for static MIMO interference channels [11][12].

4. A STACKELBERG FORMULATION OF THE NON-ORTHOGONAL CASE

We assume now that both transmitters are free to decide their power control policy. However, there is still hierarchy in the system in the sense that, for each time-slot, the primary transmitter has to choose its power level in the first place and the secondary transmitter (assumed to equipped with a cognitive radio) observes this level and reacts to it. This framework is exactly the one of a Stackelberg game since it is assumed that the primary transmitter (called the game leader) knows it is observed by a rational player (the game follower). The SINR for the first transmitter/receiver pair is:

$$SINR_1(p_1, p_2) = \frac{p_1 g_{11}}{\sigma^2 + p_2 g_{21}} := \gamma_1, \tag{14}$$

where g_{21} is the channel gain between transmitter 2 and receiver 1. For the second transmitter/receiver pair, the SINR is:

$$SINR_2(p_1, p_2) = \frac{p_2 g_{22}}{\sigma^2 + p_1 g_{12}} := \gamma_2, \tag{15}$$

where g_{12} is the channel gain between transmitter 1 and receiver 2. Using this relation, we have the powers for transmitters 1 and 2 depending on the SINRs:

$$\begin{aligned} & p_1 = \frac{\sigma^2}{g_{11}} \frac{\gamma_1 + \gamma_1\gamma_2 \frac{g_{21}}{g_{22}}}{1 - \alpha\gamma_1\gamma_2}, \quad \text{and} \quad p_2 = \frac{\sigma^2}{g_{22}} \frac{\gamma_2 + \gamma_1\gamma_2 \frac{g_{12}}{g_{11}}}{1 - \alpha\gamma_1\gamma_2} \\ & \text{with} \quad \alpha = \frac{g_{21} g_{12}}{g_{11} g_{22}}. \end{aligned} \tag{16}$$

A Stackelberg equilibrium is a vector (p_1^*, p_2^*) such that:

$$p_1^* = \arg\max_{p_1} u_1(p_1, p_2^*(p_1)), \tag{17}$$

with

$$\forall p_1, \quad p_2^*(p_1) = \arg\max_{p_2} u_2(p_1, p_2). \tag{18}$$

Note that the above expression implicitly assumes that the best-response of the follower is a singleton, which is effectively the case for the problem under study. In our Stackelberg game, the utility u_2 of the secondary transmitter/receiver pair depends on the power control scheme p_1 through the expression:

$$\begin{aligned} & \forall p_1, \quad u_2(p_1, p_2) = \\ & R_2 \int_0^{+\infty} \int_0^{+\infty} \phi_{12}(g_{12}) \phi_{22}(g_{22}) \frac{f\left(\frac{p_2 g_{22}}{\sigma^2 + p_1 g_{12}}\right)}{p_2} \mathrm{d}g_{12} \mathrm{d}g_{22}, \end{aligned} \tag{19}$$

with the energy constraint:

$$T \int_0^{+\infty} \phi_{22}(g_{22}) p_2 \mathrm{d}g_{22} \leq E_2. \tag{20}$$

In order to determine a Stackelberg equilibrium, we first have to express the best response of the follower that is, the best power control

scheme for the secondary transmitter/receiver pair, given the long term power control scheme of the primary transmitter/receiver pair.

For a given $p_1(g_{12})$, the Lagrangian L_{u_2} of u_2 is given by:

$$
\begin{aligned}
&L_{u_2}(p_1, p_2, \lambda_2) = \\
&R_2 \int_0^{+\infty} \int_0^{+\infty} \phi_{12}(g_{12}) \phi_{22}(g_{22}) \frac{f(\frac{p_2 g_{22}}{\sigma^2 + p_1 g_{12}})}{p_2} \mathrm{d}g_{12} \mathrm{d}g_{22} \\
&- \lambda_2 (T \int_0^{+\infty} \phi_{22}(g_{22}) p_2 \mathrm{d}g_{22} - E_2).
\end{aligned}
\tag{21}
$$

Proposition 1 (Optimal SINR for the secondary transmitter)**.** *The secondary transmitter has to tune its power level such that its SINR is the greatest zero of the following equation:*

$$
x f'(x) - f(x) = \frac{\lambda_2 T (\sigma^2 + p_1 g_{12})^2}{R_2 g_{22}^2} x^2. \tag{22}
$$

The proof is ready and follows the single-user case analysis, which is conducted in Sec. 3. The optimal power control scheme p_2^* of the secondary transmitter/receiver pair, depending on the power control scheme p_1 is given by:

$$
p_2^*(p_1) = \frac{\sigma^2 + p_1 g_{12}}{g_{22}} x_2(p_1), \tag{23}
$$

where $x_2(p_1)$ is the greatest solution of (22).

Now, let us the consider the case of the primary transmitter.

Proposition 2 (Optimal SINR for the primary transmitter)**.** *The primary transmitter has to tune its power level such that its SINR is the greatest zero of the following equation:*

$$
\begin{aligned}
&x f'(x) \left[1 - \alpha x_2 x - G(x)\right] - f(x) = \frac{\lambda_1 T \sigma^4}{R_1 g_{11}^2} \left(\frac{1 + \frac{g_{21}}{g_{22}} x_2}{1 - \alpha x x_2} \right)^2 x^2, \\
&\textit{with} \quad G(x) = \frac{\alpha x (1 + \frac{g_{12}}{g_{11}} x)^2 x_2}{(1 - \alpha x_2 x)^2 \frac{R_2 g_{22}^2}{2 \lambda_2 T \sigma^4} f''(x_2) - (1 + \frac{g_{12}}{g_{11}} x)^2}.
\end{aligned}
\tag{24}
$$

Proof. The leader is optimizing his utility function u_1 taking into account this best response power control scheme of the follower transmitter/receiver pair. The SINR of the leader transmitter/receiver pair, when the follower transmitter/receiver pair uses his best response power control scheme, is given by:

$$
\begin{aligned}
\mathrm{SINR}_1(p_1, p_2^*(p_1)) &= \frac{p_1 g_{11}}{\sigma^2 + p_2^*(p_1) g_{21}} \\
&= \frac{p_1 g_{11}}{\sigma^2 (1 + \frac{g_{21}}{g_{22}} x_2(p_1)) + p_1 \frac{g_{12} g_{21}}{g_{22}} x_2(p_1)}.
\end{aligned}
\tag{25}
$$

The derivative of the SINR of the leader is

$$
\begin{aligned}
&\frac{\partial \gamma_1}{\partial p_1}(p_1) = \\
&g_{11} \frac{\sigma^2 (1 + \frac{g_{21}}{g_{22}} x_2(p_1)) - p_1 \sigma^2 \frac{g_{21}}{g_{22}} x_2'(p_1) - p_1^2 \frac{g_{12} g_{21}}{g_{22}} x_2'(p_1)}{(\sigma^2 (1 + \frac{g_{21}}{g_{22}} x_2(p_1)) + p_1 \frac{g_{12} g_{21}}{g_{22}} x_2(p_1))^2}
\end{aligned}
\tag{26}
$$

Then we have

$$
\begin{aligned}
&p_1 \frac{\partial \gamma_1}{\partial p_1}(p_1) = \\
&\gamma_1(p_1) \frac{\sigma^2 (1 + \frac{g_{21}}{g_{22}} x_2(p_1)) - p_1 \sigma^2 \frac{g_{21}}{g_{22}} x_2'(p_1) - p_1^2 \frac{g_{12} g_{21}}{g_{22}} x_2'(p_1)}{\sigma^2 (1 + \frac{g_{21}}{g_{22}} x_2(p_1)) + p_1 \frac{g_{12} g_{21}}{g_{22}} x_2(p_1)}, \\
&= \gamma_1(p_1) \left(1 - \frac{p_1 \frac{g_{12} g_{21}}{g_{22}} x_2(p_1) + p_1 \sigma^2 \frac{g_{21}}{g_{22}} x_2'(p_1) + p_1^2 \frac{g_{12} g_{21}}{g_{22}} x_2'(p_1)}{\sigma^2 (1 + \frac{g_{21}}{g_{22}} x_2(p_1)) + p_1 \frac{g_{12} g_{21}}{g_{22}} x_2(p_1)} \right), \\
&= \gamma_1(p_1) \left(1 - x_2(p_1) \alpha \gamma_1(p_1) - \frac{(\sigma^2 + p_1 g_{12}) p_1 \frac{g_{21}}{g_{22}} x_2'(p_1)}{\sigma^2 (1 + \frac{g_{21}}{g_{22}} x_2(p_1)) + p_1 \frac{g_{12} g_{21}}{g_{22}} x_2(p_1)} \right), \\
&= \gamma_1(p_1) \left(1 - x_2(p_1) \alpha \gamma_1(p_1) - \frac{\sigma^2 + p_1 g_{12}}{g_{12}} \alpha x_2'(p_1) \gamma_1(p_1) \right)
\end{aligned}
\tag{27}
$$

Taking the expression of $x_2(p_1)$ we get:

$$
\begin{aligned}
&x_2' f'(x_2) + x_2 x_2' f''(x_2) - x_2' f'(x_2) = \\
&2 \frac{\lambda_2 T (\sigma^2 + p_1 g_{12})}{R_2 g_{22}^2} g_{12} x_2^2 + 2 \frac{\lambda_2 T (\sigma^2 + p_1 g_{12})^2}{R_2 g_{22}^2} x_2 x_2',
\end{aligned}
\tag{28}
$$

which yields to:

$$
x_2' f''(x_2) = 2 \frac{\lambda_2 T (\sigma^2 + p_1 g_{12})}{R_2 g_{22}^2} g_{12} x_2 + 2 \frac{\lambda_2 T (\sigma^2 + p_1 g_{12})^2}{R_2 g_{22}^2} x_2'. \tag{29}
$$

Then we get the derivative of $x_2(p_1)$:

$$
x_2'(p_1) = \frac{\frac{2 \lambda_2 T}{R_2 g_{22}^2} (\sigma^2 + p_1 g_{12}) g_{12} x_2}{f''(x_2) - \frac{2 \lambda_2 T}{R_2 g_{22}^2} (\sigma^2 + p_1 g_{12})^2}. \tag{30}
$$

Then we have:

$$
\frac{(\sigma^2 + p_1 g_{12}) x_2'(p_1)}{g_{12}} = \frac{\frac{2 \lambda_2 T}{R_2 g_{22}^2} (\sigma^2 + p_1 g_{12})^2 x_2}{f''(x_2) - \frac{2 \lambda_2 T}{R_2 g_{22}^2} (\sigma^2 + p_1 g_{12})^2}. \tag{31}
$$

Taking the expression of the power of receiver/transmitter pair 1 depending on both SINRs, we get:

$$
\sigma^2 + p_1 g_{12} = \sigma^2 \left(\frac{1 + \frac{g_{12}}{g_{11}} \gamma_1}{1 - \alpha \gamma_1 \gamma_2} \right), \tag{32}
$$

Then

$$
\frac{(\sigma^2 + p_1 g_{12}) x_2'(p_1)}{g_{12}} = \frac{(1 + \frac{g_{12}}{g_{11}} \gamma_1)^2 x_2}{(1 - \alpha \gamma_2 \gamma_1)^2 \frac{R_2 g_{22}^2}{2 \lambda_2 T \sigma^4} f''(x_2) - (1 + \frac{g_{12}}{g_{11}} \gamma_1)^2}. \tag{33}
$$

Then we have:

$$
\begin{aligned}
&p_1 \frac{\partial \gamma_1}{\partial p_1}(p_1) = \\
&\gamma_1 \left(1 - \alpha x_2 \gamma_1 - \frac{\alpha \gamma_1 (1 + \frac{g_{12}}{g_{11}} \gamma_1)^2 x_2}{(1 - \alpha x_2 \gamma_1)^2 \frac{R g_{22}^2}{2 \lambda_2 T \sigma^4} f''(x_2) - (1 + \frac{g_{12}}{g_{11}} \gamma_1)^2} \right)
\end{aligned}
\tag{34}
$$

□

By denoting x_1 the largest solution of this equation, the optimal power control scheme of the leader at the equilibrium is given by:

$$
\frac{p_1^* g_{11}}{\sigma^2 (1 + \frac{g_{21}}{g_{22}} x_2(p_1^*)) + p_1 \frac{g_{12} g_{21}}{g_{22}} x_2(p_1^*)} = x_1. \tag{35}
$$

5. NUMERICAL RESULTS

The following simulations are performed with the parameters: $T = 10^{-3}$ s, $R_1 = R_2 = 10^4$ bits/s, $\sigma^2 = 10^{-12}$ W, the channel gains g_{11} and g_{22} are assumed to follow a Rayleigh distribution of mean 10^{-10}, when needed, g_{12} and g_{21} are assumed to follow a Rayleigh distribution of mean 10^{-12} and the efficiency function used is f_a, defined in Sec. 3 with $a = 0.9$. Fig. 1 illustrates the influence of λ_E on the energy constraint in a single-user case. When λ_E is low, the optimal power control scheme is to transmit most of the time, thus the energy spent is high. On the contrary, when λ_E increases, transmission will only occurs when the channel gain is good enough, resulting in a lower energy spent. After a certain threshold, the optimal scheme is not to transmit at all.

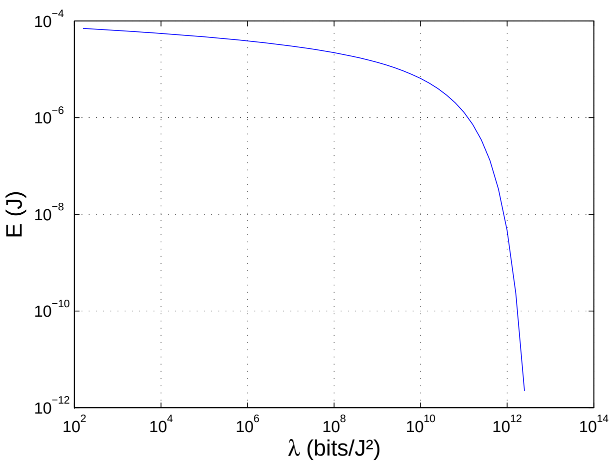

Fig. 1. Energy spent on duration T depending on λ_E.

In Fig. 2, we are in the context of Sec. 3.2. We compute the probability per time-slot that the primary link is not used and we compare it to its lower bound. It is interesting to note that this lower-bound is relatively tight to the exact probability.

Fig. 3 compares the expected utilities of Stackelberg equilibrium (Sec. 4) and the orthogonal case (Sec. 3). As we could expect, the primary link of the orthogonal case offers the best utility, but the orthogonal secondary link has the worst performance. The leader and follower of the Stackelberg case have are much more similar in terms of performance and are very clos to the performance of the primary link which makes the Stackelberg case a very efficient and fair scenario for both links. Of course, like in the single-user case, after a threshold for λ, they do not transmit at all.

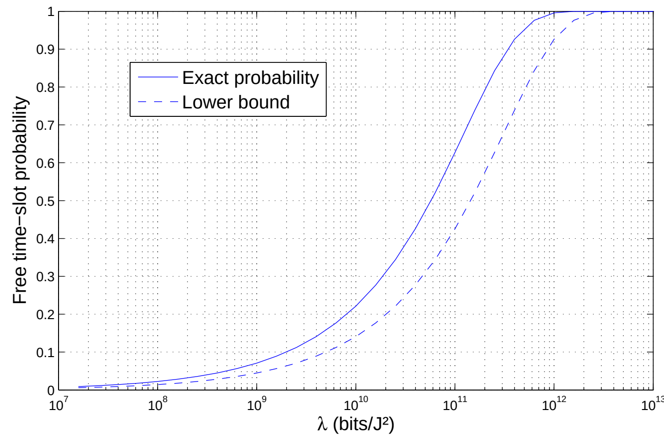

Fig. 2. Comparison of the exact probability of having free time-slot with the proposed lower bound of this probability.

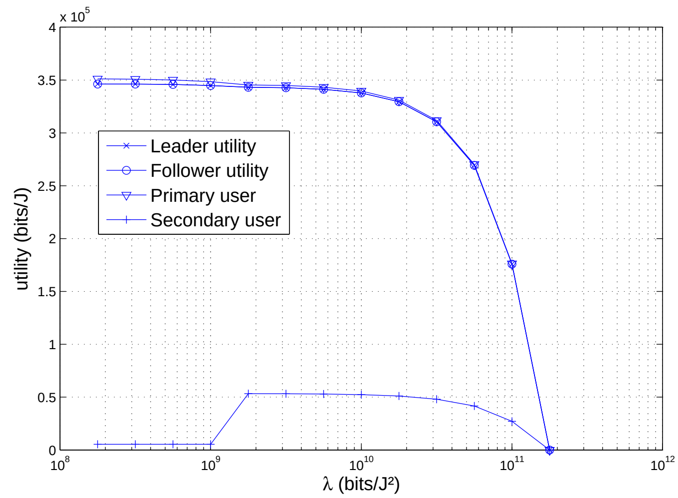

Fig. 3. Comparison of the expected utilities of Stackelberg equilibrium and the orthogonal case depending on λ. In this particular case, $\lambda_1 = \lambda_2 = \lambda$.

In particular, Fig. 4 shows the optimal power profile of the leading transmitter w.r.t. the channels gains g_{11} and g_{22} when $\lambda = 10^10$ bits/J^2. It is clear that for low values of g_{11}, the optimal policy is not to transmit. Then we distinguish two zones of interest:

- when both g_{11} and g_{22} are good, the transmitter uses most of its power for a relatively high value of g_{11},
- when only g_{11} is good, we can see that the transmitter uses most of its power for a lower value of g_{11} as it is not likely to facing interference from the following transmitter in this zone.

6. CONCLUSION AND PERSPECTIVES

In this paper, it is shown how a long-term energy constraint modifies the behavior of a transmitter in terms of power control policy. In

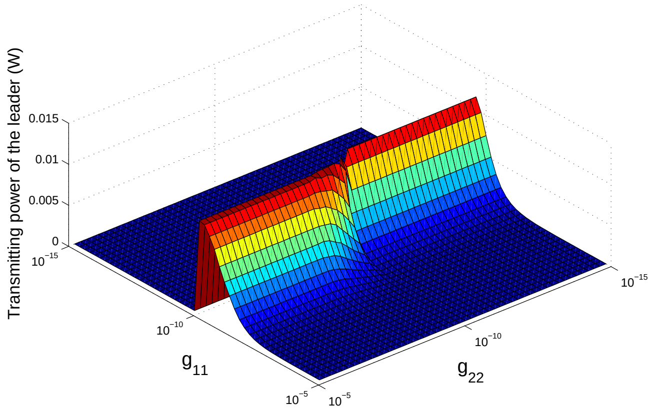

Fig. 4. Power profile of the leading transmitter w.r.t. g_{11} and g_{22} in the two-player Stackelberg case.

contrast with related works such as [4][5][6], a transmitter does not always transmit when it is subject to such a constraint. This shows that when implementing its best power control policy, a primary link does not exploit all the available time-slots. The probability of having a free time-slot for the secondary link can be lower bounded in a reasonably tight manner and shown to be non-negligible in general. As a second step, a scenario where the secondary link can interfere on the primary link is analyzed. The problem is formulated as a Stackelberg game where the primary transmitter is the leader and the secondary transmitter is the follower. An equilibrium in this game is shown to exist for typical conditions on the efficiency function $f(x)$. Interestingly, the fact that the transmitters have a long-term energy constraint can make the system more efficient since this incites users to interfere less; indeed simulations show the existence of a value of an energy budget which maximizes the users's utilities. While the power control schemes at the equilibrium can be determined, the corresponding equations have a drawback: the power control scheme of a given user does not only rely on the knowledge of its individual channel gain but also on the other channel gains. This shows the relevance of improving the proposed work by designing more distributed power control policies. Additionally, the proposed scenarios included one primary link and one secondary link. When several cognitive transmitters are present, there is a competition between the secondary transmitters for exploiting the resources left by the primary link.

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